Hereditary orders in the quotient ring of a skew polynomial ring

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Abstract

Let $K$ be a field, and let $\sigma$ be an automorphism of $K$ of finite order, say $n$. One can form a skew polynomial ring $K[X, \sigma]$ over $K$ with the usual rules of multiplication defined by the commutation rule: $Xn = \sigma(n)X \forall n \in K$. Let $K(X, \sigma)$ denote the skew field of quotients of $K(X, \sigma)$. If $F$ is the fixed field of $\sigma$, then $K(X, \sigma)$ is a cyclic algebra of degree $n$ with center $F(X^n)$. If $V$ is a valuation ring of $F(X^n)$ containing $F$, and $S$ is the integral closure of $V$ in $K(X^n)$, then any order of $K(X, \sigma)$ with center $V$ can be written as a "crossed-product $V$-algebra":

$$A_f = \sum_{i=0}^{n-1} Sx_i,$$

with the multiplication rule $x_i x_j = \sigma^j(x_i)x_j$ for all $s \in S, 0 \leq i < n$ and $x_i x_0 = f(\sigma^i, \sigma^{-1})x_{i+1}$, where $f : G \times G \rightarrow S \setminus \{0\}$ is some normalized 2-cocycle, and $G$ is the Galois group of the cyclic extension $K(X^n)/F(X^n)$.

Let $H = \{\sigma^i | f(\sigma^i, \sigma^{-1}) \in U(S)\}$, where $U(S)$ denotes the group of the multiplicative units of the ring $S$. Then $H$ is a subgroup of $G$. On $G/H$, one can define a partial ordering by the rule

$$\sigma^i H \leq \sigma^j H \text{ if } f(\sigma^i, \sigma^{-1}) \in U(S).$$

Then $\leq$ is well-defined, and depends only on the cohomology class of $f$ over $S$. Further, $H$ is the unique least element. We call this partial ordering on $G/H$ the graph of $f$.

The aim of the talk is to determine the conditions on the graph of $f$ that would guarantee that $A_f$ is a hereditary order.

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