A note on hyperbolic equations with nonlocal boundary and Dirichlet-Neumann conditions

Özgür Yıldırım

Department of Mathematics, Uludağ University
16500 Görükle, Bursa, Turkey
yozgur@uludag.edu.tr

Abstract

In present paper joint with Prof. Allaberen Ashyraliev, the nonlocal boundary value problem

\[
\begin{aligned}
\frac{\partial^2 u(t,x)}{\partial t^2} - \sum_{i=1}^{m} (\alpha_i(x)u_{x_i})_{x_i} + \sigma u &= f(t,x), \\
x = (x_1, \ldots, x_m) &\in \Omega, \quad 0 < t < 1, \\
u(0,x) &= \sum_{j=1}^N \alpha_j u(\lambda_j, x) + \varphi(x), \\
u_t(0,x) &= \sum_{k=1}^K \beta_k u_t(\lambda_k, x) + \psi(x), \quad x \in \overline{\Omega}, \\
x \in \overline{\Omega}, \quad u(t,x) &= 0, \quad x \in S_1, \\
\frac{\partial u(t,x)}{\partial n} &= 0, \quad 0 \leq t \leq 1, x \in S_2,
\end{aligned}
\]

(0.1)

for the multidimensional hyperbolic equation is considered. Here \( \Omega \) be the unit open cube in the \( m \)-dimensional Euclidean space \( \mathbb{R}^m \) \( x = (x_1, \ldots, x_m) : \)

\[ 0 < x_j < 1, 1 \leq j \leq m \] with boundary \( S = S_1 \cup S_2, \quad \overline{\Omega} = \Omega \cup S \).

The first and second order of accuracy difference schemes for the numerical solution of hyperbolic equations with nonlocal boundary and Dirichlet-Neumann conditions are presented. The stability estimates for the solutions of the difference schemes are obtained. A procedure of modified Gauss elimination method is used for solving these difference schemes in the case of one dimensional hyperbolic equation.

References


2000 Mathematics Subject Classification. 65N12: 65M12: 65J10

Key words and phrases. hyperbolic equation, nonlocal boundary value problems, difference schemes, stability